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$$= \int_0^a \int_0^{(a^{1/2}-x^{1/2})^2} [(a^{1/2} - x^{1/2})^2 - 2(a^{1/2} - x^{1/2})y^{1/2} + y]dydx,$$

$$= 1/6 \int_0^a [a^2 - 4a^{3/2}x^{1/2} + 6ax - 4a^{1/2}x^{3/2} + x^2]dx,$$

$$= 1/6 \left[a^3 - 8/3a^3 + 3a^3 - 8/5a^3 + \frac{a^3}{3}\right] = \frac{a^3}{90}.$$

II. Notes by R. C. Archibald, Brown University.

The integral here arising is a particular case of one considered by Lejeune Dirichlet in 1839.[1] If $V = \int x^{a-1}y^{b-1}z^{c-1} \cdots dxdydz \cdots$, considering all positive values of $x, y, z, \cdots$ such that

$$\left(\frac{x}{\alpha}\right)^p + \left(\frac{y}{\beta}\right)^q + \left(\frac{z}{\gamma}\right)^r + \cdots < 1$$

the constants $a, b, c, \cdots, p, q, r, \cdots, \alpha, \beta, \gamma, \cdots$ being also positive, then

$$V = \frac{\alpha^a\beta^b\gamma^c \cdots}{pqr \cdots} \frac{\Gamma\left(\frac{a}{p}\right)\Gamma\left(\frac{b}{q}\right)\Gamma\left(\frac{c}{r}\right)\cdots}{\Gamma\left(1 + \frac{a}{p} + \frac{b}{q} + \frac{c}{r} + \cdots\right)}.$$

For the surface $(x/a)^{1/n} + (y/b)^{1/n} + (z/c)^{1/n} = 1$, the volume in the first octant would be $abc(n!)^3/(3n)!$ When $n = 7/2$ we have a result given[2] in 1883

$$V = \frac{abc}{(2/7)^3} \cdot \frac{[\Gamma(7/2)]^3}{\Gamma(23/2)}.$$

The values, for $n = 3/2$ and $n = 1/2$ were given in Williamson, *Elementary Treatise on the Integral Calculus*, 6 ed., 1891, pp. 289–290. The result for the surface $(x/a)^2 + (y/b)^2 + (z/c)^4 = 1$, given in Todhunter, *A Treatise on the Integral Calculus*, 4 ed., 1874, p. 186, follows at once from the Dirichlet formula above.

2852 [1920, 377]. Proposed by D. H. RICHERT, Bethel College, Newton, Kans.

What is the radius of a cylinder inscribed in a right cone, radius of base $R = 5$ inches, and altitude $h = 18$ inches, the volume of the cylinder to be $1/n$ $(= 3/4)$ that of the cone?

Solution by H. S. Uhler, Yale University.

Let V denote the volume of the cone, and let r, v, and z denote, respectively, the radius, the volume, and the altitude of the cylinder. Then

$$V = \tfrac{1}{3}\pi hR^2, \qquad v = \pi zr^2, \qquad \text{and} \qquad v = \frac{1}{n}V;$$

hence,

$$zr^2 = \frac{hR^2}{3n}. \tag{1}$$

From the similar right triangles obtained by passing a plane through the common axis of the cone and cylinder we find

$$z = \frac{h(R - r)}{R}. \tag{2}$$

These two equations are homogeneous in h and z, and also in R and r; therefore they determine the ratio of the altitudes and the ratio of the radii independently as functions of n alone.

Substituting from (2) for z in equation (1) we obtain

$$r^3 - Rr^2 + R^3/(3n) = 0.$$

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[1] *Comptes Rendus . . . de l'Académie des Sciences*, vol. 8, p. 156; also in *Journal de Mathématiques Pures et Appliquées*, vol. 4, p. 168.

[2] *Mathematical Questions with their solutions from the "Educational Times,"* vol. 38, p. 104.

Let $\rho \equiv r/R$ so that the cubic becomes

$$\rho^3 - \rho^2 + 1/(3n) = 0. \tag{3}$$

Since the constant term of this equation is positive there is always one real negative root, and this is inadmissible. Therefore the remaining roots must be real in order that a positive root may exist. The derivative of the left-hand member of (3) is $\rho(3\rho - 2)$ which vanishes for $\rho = 0$ and $\rho = 2/3$. The expression for the derivative shows that there is a maximum for $\rho = 0$, and a minimum for $\rho = 2/3$; the value of the minimum is $1/3(1/n - 4/9)$ and hence if $1/n > 4/9$ there is only one real root, the negative root already mentioned.

Consequently the given value $1/n = 3/4$ conflicts with the preceding condition and makes the problem impossible.

Also solved by T. M. Blakslee, L. A. Eastburn, R. A. Johnson, R. H. Marshall, J. Q. McNatt, Arthur Pelletier, and A. V. Richardson.

2855 [1920, 377]. Proposed by J. L. RILEY, Stephenville, Texas.

Show that the circle of curvature at any point of the ellipse cannot pass through the centre unless the eccentricity be greater than $1/\sqrt{2}$.

I. Solution by A. V. Richardson, Bishop's College, Lennoxville, Quebec.

In the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b),$$

the normal at the point $(a \cos \theta, b \sin \theta)$ is

$$\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2. \tag{1}$$

To find the coördinates of the center of curvature, we solve (1) and its derivative with respect to θ,

$$\frac{ax \sin \theta}{\cos^2 \theta} + \frac{by \cos \theta}{\sin^2 \theta} = 0,$$

giving

$$x = \frac{a^2 - b^2}{a} \cos^3 \theta; \qquad y = -\frac{a^2 - b^2}{b} \sin^3 \theta. \tag{2}$$

The (radius of curvature)^2 is, of course, the (distance)^2 between this point and $(a \cos \theta, b \sin \theta)$ *i.e.*,

$$\frac{(a^2 \sin^2 \theta + b^2 \cos^2 \theta)^3}{a^2 b^2}.$$

If we put the radius of curvature equal to the distance of the center of curvature from the origin we shall get after some reductions

$$(a^2 - 2b^2) \cos^2\theta = (2a^2 - b^2) \sin^2\theta. \tag{3}$$

Hence, for real values of θ, we must have $a^2 > 2b^2$, *i.e.*,

$$a^2 > 2a^2 (1 - e^2) \qquad \text{or} \qquad e > \frac{1}{\sqrt{2}}.$$

II. Remarks by Otto Dunkel, Washington University.

If we determine the condition that the perpendicular bisector of the segment from the origin to the point $(a \cos \theta, b \sin \theta)$ shall pass through the center of curvature we shall get equation (3) more easily.

Or we may express this by saying that the center of curvature projects into the middle point of the segment. This condition takes the form $2(x_1 x_c + y_1 y_c) - (x_1^2 + y_1^2) = 0$, which reduces to the same equation (3).

We find also that e may equal $1/\sqrt{2}$. In the ellipse for which $e = 1/\sqrt{2}$ the radius of curvature at the extremity of the major axis is equal to $a/2$.

Also solved by A. M. Harding, William Hoover, and H. S. Uhler.